7.4 Applications

1. Let \( x \) and \( y \) represent the two numbers. The system of equations is:
   \[
   \begin{align*}
   x + y &= 25 \\
   y &= 5 + x
   \end{align*}
   \]
   Substituting into the first equation:
   \[
   x + (5 + x) = 25 \\
   2x + 5 = 25 \\
   2x = 20 \\
   x = 10 \\
   y = 5 + 10 = 15
   \]
   The two numbers are 10 and 15.

3. Let \( x \) and \( y \) represent the two numbers. The system of equations is:
   \[
   \begin{align*}
   x + y &= 15 \\
   y &= 4x
   \end{align*}
   \]
   Substituting into the first equation:
   \[
   x + 4x = 15 \\
   5x = 15 \\
   x = 3 \\
   y = 4(3) = 12
   \]
   The two numbers are 3 and 12.

5. Let \( x \) represent the larger number and \( y \) represent the smaller number. The system of equations is:
   \[
   \begin{align*}
   x - y &= 5 \\
   x &= 2y + 1
   \end{align*}
   \]
   Substituting into the first equation:
   \[
   2y + 1 - y = 5 \\
   y + 1 = 5 \\
   y = 4 \\
   x = 2(4) + 1 = 9
   \]
   The two numbers are 4 and 9.
7. Let \( x \) and \( y \) represent the two numbers. The system of equations is:
   \[
   y = 4x + 5
   \]
   \[
   x + y = 35
   \]
Substituting into the second equation:
   \[
   x + 4x + 5 = 35
   \]
   \[
   5x + 5 = 35
   \]
   \[
   5x = 30
   \]
   \[
   x = 6
   \]
   \[
   y = 4(6) + 5 = 29
   \]
The two numbers are 6 and 29.

9. Let \( x \) represent the amount invested at 6% and \( y \) represent the amount invested at 8%.
The system of equations is:
   \[
   x + y = 20000
   \]
   \[
   0.06x + 0.08y = 1380
   \]
Multiplying the first equation by \(-0.06\):
   \[
   -0.06x + 0.06y = -1200
   \]
   \[
   0.06x + 0.08y = 1380
   \]
Adding the two equations:
   \[
   0.02y = 180
   \]
   \[
   y = 9000
   \]
Substituting into the first equation:
   \[
   x + 9000 = 20000
   \]
   \[
   x = 11000
   \]
Mr. Wilson invested $9,000 at 8% and $11,000 at 6%.

11. Let \( x \) represent the amount invested at 5% and \( y \) represent the amount invested at 6%.
The system of equations is:
   \[
   x = 4y
   \]
   \[
   0.05x + 0.06y = 520
   \]
Substituting into the second equation:
   \[
   0.05(4y) + 0.06y = 520
   \]
   \[
   0.20y + 0.06y = 520
   \]
   \[
   0.26y = 520
   \]
   \[
   y = 2000
   \]
   \[
   x = 4(2000) = 8000
   \]
She invested $8,000 at 5% and $2,000 at 6%.
13. Let \( x \) represent the number of nickels and \( y \) represent the number of quarters. The system of equations is:
\[
\begin{align*}
x + y &= 14 \\
0.05x + 0.25y &= 2.30
\end{align*}
\]
Multiplying the first equation by \(-0.05\):
\[
\begin{align*}
-0.05x - 0.05y &= -0.7 \\
0.05x + 0.25y &= 2.30
\end{align*}
\]
Adding the two equations:
\[
0.20y = 1.6
\]
\[
y = 8
\]
Substituting into the first equation:
\[
x + 8 = 14
\]
\[
x = 6
\]
Ron has 6 nickels and 8 quarters.

15. Let \( x \) represent the number of dimes and \( y \) represent the number of quarters. The system of equations is:
\[
\begin{align*}
x + y &= 21 \\
0.10x + 0.25y &= 3.45
\end{align*}
\]
Multiplying the first equation by \(-0.10\):
\[
\begin{align*}
-0.10x - 0.10y &= -2.10 \\
0.10x + 0.25y &= 3.45
\end{align*}
\]
Adding the two equations:
\[
0.15y = 1.35
\]
\[
y = 9
\]
Substituting into the first equation:
\[
x + 9 = 21
\]
\[
x = 12
\]
Tom has 12 dimes and 9 quarters.
17. Let $x$ represent the liters of 50% alcohol solution and $y$ represent the liters of 20% alcohol solution. The system of equations is:

$$x + y = 18$$
$$0.50x + 0.20y = 0.30(18)$$

Multiplying the first equation by $-0.20$:

$$-0.20x - 0.20y = -3.6$$
$$0.50x + 0.20y = 5.4$$

Adding the two equations:

$$0.30x = 1.8$$
$$x = 6$$

Substituting into the first equation:

$$6 + y = 18$$
$$y = 12$$

The mixture contains 6 liters of 50% alcohol solution and 12 liters of 20% alcohol solution.

19. Let $x$ represent the gallons of 10% disinfectant and $y$ represent the gallons of 7% disinfectant. The system of equations is:

$$x + y = 30$$
$$0.10x + 0.07y = 0.08(30)$$

Multiplying the first equation by $-0.07$:

$$-0.07x - 0.07y = -2.1$$
$$0.10x + 0.07y = 2.4$$

Adding the two equations:

$$0.03x = 0.3$$
$$x = 10$$

Substituting into the first equation:

$$10 + y = 30$$
$$y = 20$$

The mixture contains 10 gallons of 10% disinfectant and 20 gallons of 7% disinfectant.
21. Let \( x \) represent the number of adult tickets and \( y \) represent the number of kids tickets. The system of equations is:

\[
\begin{align*}
x + y &= 70 \\
5.50x + 4.00y &= 310
\end{align*}
\]

Multiplying the first equation by \(-4\):

\[
\begin{align*}
-4.00x - 4.00y &= -280 \\
5.50x + 4.00y &= 310
\end{align*}
\]

Adding the two equations:

\[
1.5x = 30 \\
x = 20
\]

Substituting into the first equation:

\[
20 + y = 70 \\
y = 50
\]

The matinee had 20 adult tickets sold and 50 kids tickets sold.

23. Let \( x \) represent the width and \( y \) represent the length. The system of equations is:

\[
\begin{align*}
x &+ 2y = 96 \\
y &= 2x
\end{align*}
\]

Substituting into the first equation:

\[
\begin{align*}
2x + 2(2x) &= 96 \\
2x + 4x &= 96 \\
6x &= 96 \\
x &= 16 \\
y &= 2(16) = 32
\end{align*}
\]

The width is 16 feet and the length is 32 feet.

25. Let \( x \) represent the number of $5 chips and \( y \) represent the number of $25 chips. The system of equations is:

\[
\begin{align*}
x &+ y = 45 \\
5x + 25y &= 465
\end{align*}
\]

Multiplying the first equation by \(-5\):

\[
\begin{align*}
-5x - 5y &= -225 \\
5x + 25y &= 465
\end{align*}
\]

Adding the two equations:

\[
20y = 240 \\
y = 12
\]

Substituting into the first equation:

\[
\begin{align*}
x + 12 &= 45 \\
x &= 33
\end{align*}
\]

The gambler has 33 $5 chips and 12 $25 chips.
27. Let \( x \) represent the number of shares of $11 stock and \( y \) represent the number of shares of $20 stock. The system of equations is:

\[
\begin{align*}
  x + y &= 150 \\
  11x + 20y &= 2550
\end{align*}
\]

Multiplying the first equation by \(-11\):

\[
\begin{align*}
  -11x - 11y &= -1650 \\
  11x + 20y &= 2550
\end{align*}
\]

Adding the two equations:

\[
9y = 900
\]

\[
y = 100
\]

Substituting into the first equation:

\[
x + 100 = 150
\]

\[
x = 50
\]

She bought 50 shares at $11 and 100 shares at $20.

29. Reducing the rational expression:

\[
\frac{x^2 - x - 6}{x^2 - 9} = \frac{(x - 3)(x + 2)}{(x + 3)(x - 3)} = \frac{x + 2}{x + 3}
\]

31. Performing the operations:

\[
\frac{x^2 - 25}{x + 4} \cdot \frac{2x + 8}{x^2 - 9x + 20} = \frac{(x + 5)(x - 5)}{x + 4} \cdot \frac{2(x + 4)}{(x - 4)(x - 5)} = \frac{2(x + 5)(x - 5)(x + 4)}{(x - 4)(x - 5)(x - 5)} = \frac{2(x + 5)}{x - 4}
\]

33. Performing the operations:

\[
\frac{x}{x^2 - 16} + \frac{4}{x^2 - 16} = \frac{x + 4}{(x + 4)(x - 4)} = \frac{1}{x - 4}
\]

35. Simplifying the complex fraction:

\[
\frac{1}{x^2} - \frac{25}{x^2} = \frac{1 - 25}{x^2} = \frac{1}{x^2} - \frac{8}{x^2} + \frac{15}{x^2} = \frac{(x + 5)(x - 5)}{(x - 5)(x - 3)} = \frac{x + 5}{x - 3}
\]

37. Multiplying each side of the equation by \( x^2 - 9 = (x + 3)(x - 3) \):

\[
(x + 3)(x - 3) \left( \frac{x}{x^2 - 9} - \frac{3}{x - 3} \right) = (x + 3)(x - 3) \cdot \frac{1}{x + 3}
\]

\[
\begin{align*}
  x - 3(x + 3) &= x - 3 \\
  x - 3x - 9 &= x - 3 \\
  -2x - 9 &= x - 3 \\
  -3x &= 6 \\
  x &= -2
\end{align*}
\]

Since \( x = -2 \) checks in the original equation, the solution is \( x = -2 \).
39. Let \( t \) represent the time to fill the pool with both pipes open. The equation is:

\[
\frac{1}{8} - \frac{1}{12} = \frac{1}{t}
\]

\[
24t \left( \frac{1}{8} - \frac{1}{12} \right) = 24t \cdot \frac{1}{t}
\]

\[
3t - 2t = 24
\]

\[
t = 24
\]

It will take 24 hours to fill the pool with both pipes left open.

41. The variation equation is \( y = Kx \). Finding \( K \):

\[
8 = K \cdot 12
\]

\[
K = \frac{2}{3}
\]

So \( y = \frac{2}{3}x \). Substituting \( x = 36 \): \( y = \frac{2}{3}(36) = 24 \)