# Lesson 5.4 Variation

## Activity 1 Direct Variation

**1.** Complete the statements about direct variation.

**a.** Definition: $y$ is **proportional** to $x$, or **varies directly** with $x$, if the ratio $\frac{y}{x}$ is ______________ .

**b.** From this fact we can write a formula for $y$ in terms of $x$: ____________ .

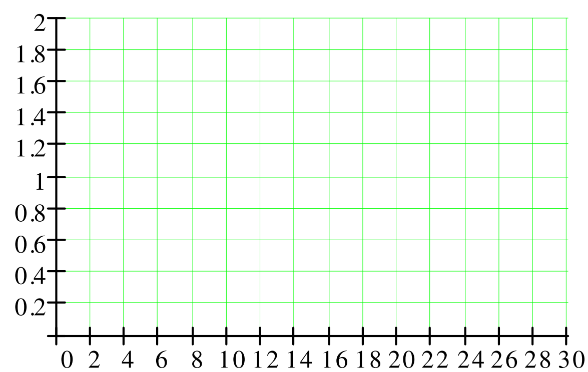
**c.** Use your answer to (**b**) to describe the graph of a direct variation:

**d.** The **constant of variation** is just the _____________ of the graph.

**2.** Delbert's credit card statement lists three purchases he made while on a business trip in the midwest. His company's accountant would like to know the sales tax rate on the purchases.

| Price of item | 18 | 28 | 12 |
|---|---|---|---|
| Tax | 1.17 | 1.82 | 0.78 |
| Tax/Price | | | |

**a.** Compute the ratio of the tax to the price of each item. Is the tax proportional to the price? What is the tax rate?




**b.** Express the tax, $T$, as a function of the price, $p$, of the item.

**c.** Sketch a graph of the function.

**d.** What is the slope of the graph?

**3.** Look back at the example above to answer the questions.

**a.** How can you recognize a direct variation from a table of values?

**b.** How can you find the constant of variation?

**General strategy for variation problems:**

**Step 1** First, use the values given in the problem to find the constant of variation. Then you can write a formula for the function.

**Step 2** Now use the formula to answer the questions.

**4.** The weight of an object on the moon varies directly with its weight on earth. A person who weighs 150 pounds on earth would weigh only 24.75 pounds on the moon.

**a.** Find a formula that gives the weight $m$ of an object on the moon in terms of its weight $w$ on earth.

**b.** Complete the table.

| $w$ | 100 | 150 | 200 | 400 |
|---|---|---|---|---|
| $m$ | | | | |

**c.** Use the table to choose a suitable window and graph your function on your calculator. Sketch the graph in the space above, and label the window settings.

**d.** How much would a person weigh on the moon if she weighs 120 pounds on earth?

**e.** A piece of rock weighs 50 pounds on the moon. How much will it weigh back on earth?

**f.** Label the points on your graph that correspond to your answers for (**d**) and (**e**).

## Activity 2 Direct Variation with a Power

**1.** Complete the statements about direct variation.

**a.** The formula for direct variation with a power is ____________________ .

**b.** From that formula, we see that the ratio ________________ is constant.

**c.** The graph of a direct variation always passes through ______________ .

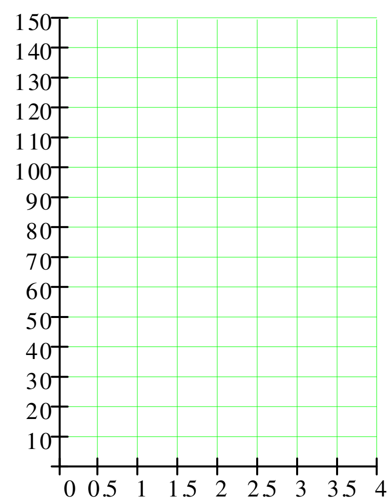
**2.** At constant acceleration from rest, the distance traveled by a racecar is proportional to the square of the time elapsed. The highest recorded road-tested acceleration is $0$ to $60$ miles per hour in $3.07$ seconds, which produces the following data.

| Time (seconds) | $2$ | $2.5$ | $3$ |
|---|---|---|---|
| Distance (feet) | $57.32$ | $89.563$ | $128.97$ |
| Distance/Time$^2$ | | | |

**a.** Compute the ratios of the distance traveled to the **square** of the time elapsed. What is the constant of proportionality?

**b.** Express the distance traveled, $d$, as a function of time in seconds, $t$.

**c.** Sketch a graph of the function.




**3.** Look back at the example above to answer the questions.

**a.** In part (**a**), note that you computed the ratio $\dfrac{d}{t^2}$ (**not** the ratio $\dfrac{d}{t}$). Why?

**b.** What sort of graph do you get in part (**c**)?

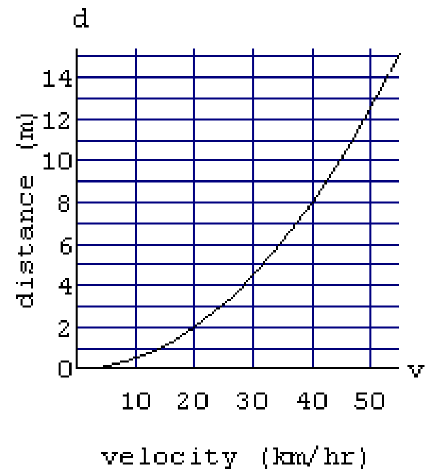
**4.** The faster a car moves, the more difficult it is to stop. The graph shows the distance, $d$, required to stop a car as a function of its velocity, $v$, before the brakes were applied.

**a.** Find a formula for $d$ as a function of $v$.

Hints: 1) How do you know from the graph that $d = kv^2$ (instead of $d = kv$) ?

2) Use a point on the graph to find the value of $k$, and then write the formula.

**b.** How fast was the car moving if it took 50 meters to stop?




**Activity 3 Inverse Variation**

**1.** Complete the statements about inverse variation.

**a.** The formula for inverse variation is ________________ .

**b.** An inverse variation is an example of a __________________ function.

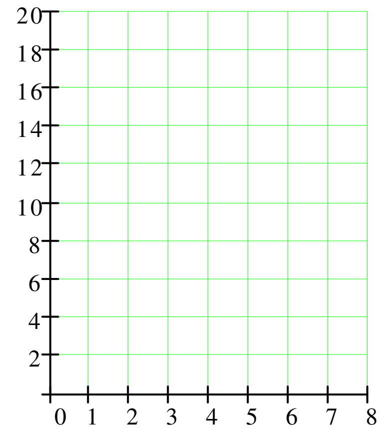
**c.** The graph of an inverse variation has a _____________________ at $x = 0$.

**d.** If $y$ varies inversely with $x$, so that $y = \frac{k}{x}$ , what is true about the product of the variables?

**2.** The marketing department for a paper company is testing wrapping paper rolls in various dimensions to see which shape consumers prefer. All the rolls contain the same amount of wrapping paper.

| Width (feet) | $2$ | $2.5$ | $3$ |
|---|---|---|---|
| Length (feet) | $12$ | $9.6$ | $8$ |
| Length · Width | | | |

**a.** Compute the product of the length and width for each roll of wrapping paper. What is the constant of inverse proportionality?




**b.** Express the length, $L,$ of the paper as a function of the width, $w,$ of the roll.

**c.** Sketch a graph of the function. Which basic graph does your answer resemble?

**3.** Ocean temperatures are generally colder at the greater depths. The table shows the temperature of the water as a function of depth.

**a.** How do you know from the table that this is an inverse variation?

| Depth (m) | Temperature (° C) |
|---|---|
| 100 | 30 |
| 200 | 15 |
| 300 | 10 |
| 400 | 7.5 |

**b.** Find a formula for $T$, temperature, in terms of $D$, depth.

**c.** What is the ocean temperature at a depth of 500 meters?

## Wrap-Up

In this Lesson, we worked on the following skills and goals related to functions:

- Recognize direct and inverse variation from a table of values
- Find the constant of variation and write an equation
- Sketch the graph of a direct or inverse variation
- Use scaling in inverse or direct variation

### Check Your Understanding

**1.** How can you recognize direct variation from an equation, a graph, or a table of values?
**2.** What ratio should you compute to see if $y$ varies directly with $x^3$?
**3.** Is every decreasing function an inverse variation? Explain.
**4.** How do you know that the table in Activity 3 represents inverse variation?